Potential Vorticity-Based Interpretation of the Evolution of The Greenhouse Low, 3 February 1991

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Abstract

The explosive synoptic-scale “Greenhouse Low” that hit Iceland on 3 February 1991 has been examined in the Ertel potential vorticity (PV) framework, with the aim of understanding the dynamical and physical interactions that took place during this event. Three positive PV anomalies were investigated: 1) a surface thermal anomaly, 2) a lower-tropospheric, diabatically produced PV anomaly, and 3) a PV anomaly propagating along the tropopause. Through piecewise inversions, we attempt to quantify the contributions of these three anomalies to the total geopotential field. By using a specific, quasi-linear procedure, the total geopotential field can be retrieved from the sum of the mean field and all PV anomalies (positive and negative). The piecewise inversions were performed at different times and for different simulations using HIRLAM. This allows us to draw a comprehensive picture of the time evolution of the cyclone in terms of the roles played by different dynamical and physical processes.

Initially, the surface thermal anomaly on the southeastern flank of the baroclinic region was crucial in spinning up the cyclone. Latent heating due to frontal ascent then took over as the most important contributor to the cyclone deepening. The associated PV anomaly intensified explosively between 06 UTC 2 February and 00 UTC 3 February. The upper-level PV anomaly also played an important role, especially in the later stages, between 06 and 12 UTC 3 February.

The results found here seem to confirm the findings from a previous, less quantitative study, on the causes for the failure of operational models in predicting this storm successfully. Too weak surface baroclinicity early on seems to prevent the cyclone from interacting constructively with the more robust upper-level anomaly.
1 Introduction

The area around Iceland is one of the most favoured regions in the world for cyclonic activity. This includes both synoptic-scale cyclones and meso-cyclones, e.g., polar lows. This large activity is caused by several factors, including strong gradients in sea-surface temperature and lee effects due to Greenland. A series of studies is currently under way, with the aim of better understanding the causes for different types of cyclones in this region, as well as the role of dynamical and physical processes in their development.

In the first of these studies, the so-called Greenhouse Low of 2–3 February 1991 was investigated through a combination of model simulations and careful re-analysis of surface observations and satellite data, see Kristjánsson and Thorsteinsson (1994, hereafter KT1) and Kristjánsson and Thorsteinsson (1995, hereafter KT2). The cyclone was not unusually deep but it was characterized by very rapid deepening and strong pressure gradients, putting it in the ranks of the most destructive storms to hit Iceland in this century. The measured pressure tendency reached +30.4 hPa/3h at one station (KT1). We now return to this same cyclone, using the potential vorticity perspective (see, e.g., Hoskins et al. 1985) to investigate the cyclone evolution in more detail. The morphology of the primary features in Ertel’s potential vorticity (EPV) field will be documented. An EPV piecewise inversion algorithm is used to determine objectively the relative importance of different EPV sources for the evolution. The method has been described by Davis and Emanuel (1991), hereafter DE. This represents an improvement over inversion techniques based on quasi-geostrophic potential vorticity (QGPV), see, e.g., Hakim et al. (1996), since we here retain the non-linear properties of EPV.

At 6-hour intervals, we subjectively select three PV anomalies that are believed to be of the greatest importance for the evolution of the low-level cyclone. We then perform the inversion piecewise, using a method described by Davis (1992), which ensures that “the pieces add up”, to yield the total field. By combining this procedure with the results of the sensitivity experiments as described by KT2, we can deduce a fairly comprehensive picture of the role of different mechanisms in the evolution of the cyclone. In the near future, it is intended to apply this method to other cases as well.

In the last few years, many authors have used PV diagnostics in various forms to assess different aspects of extratropical cyclones. For instance, Grønás (1995) showed how a traveling upper-level PV anomaly first created an explosive cyclone in the North Atlantic, and then how a low-level PV anomaly, created by latent heat release, caused an intense back-bent frontal development, which contributed strongly to the devastating winds that occured. Other studies along similar lines have been conducted by, e.g., Reader and Moore (1995). Recently, Wu and Emanuel (1995a, 1995b) published two articles, where hurricane movement was investigated using the methodology of DE and Davis (1992), that is also used here.

The synoptic description as well as analysis of the evolution and structure of the cyclone have been presented by KT2. The diagnostic methods used in this study are described in section 2. Section 3 describes the results of applying these methods to our case, rendering the time evolution of the various PV features during the rapid deepening phase of the storm. Following an assessment of the methodology and the results found in section 4, the conclusions are stated in section 5.
2 Potential vorticity diagnostics

2.1 Background

The significance of potential vorticity (PV) as a diagnostic quantity in meteorology is largely due to its conservation properties. As shown originally by Ertel (1942), PV is conserved following the motion of a particle in adiabatic, frictionless flow. This leads to many interesting effects. For instance, air descending from the stratosphere, where the static stability is large, will tend to acquire cyclonic vorticity as it enters the troposphere, where the stability is much weaker, due to the conservation of PV. As pointed out by Hoskins et al. (1985) this can have large implications for cyclone deepening, since tropopause foldings with associated descent of air from the stratosphere to the troposphere are indeed an observed feature of many synoptic-scale cyclones (Reed, 1955; Shapiro, 1970; Uccellini, 1990).

2.2 The PV inversion system

Following Ertel (1941), we define Ertel’s PV (EPV) as:

\[ q = \frac{1}{\rho} \vec{\eta} \cdot \nabla \theta \]  

(1)

Here \( \rho \) denotes density and \( \vec{\eta} \) is the absolute vorticity vector, while \( \theta \) is potential temperature. We can now express a very important aspect of PV, namely the “invertibility principle” which states that, given appropriate balance requirements as well as boundary conditions, all the dynamical fields can be uniquely obtained from the PV field. Following DE, the above equation can be rewritten in spherical coordinates as:

\[ q = -\frac{g \kappa \pi}{p} \left( \frac{\partial \theta}{\partial \pi} - \frac{1}{a \cos \phi} \frac{\partial v}{\partial \lambda} \frac{\partial \theta}{\partial \lambda} + \frac{1}{a \partial \pi \partial \phi} \right) \]  

(2)

where \( \lambda \) denotes longitude, \( \phi \) latitude and \( a \) is the radius of the earth. The westerly and southerly wind components are given as, respectively, \( u \) and \( v \), while \( \pi \) denotes the Exner function \( c_p \frac{T}{\rho} \), and \( \kappa = 0.286 \). We now rewrite (2) in terms of geopotential height, \( \Phi \), and a non-divergent streamfunction, \( \Psi \). Further, we perform a scale analysis as described by DE, yielding:

\[ q = \frac{g \kappa \pi}{p} \left[ (f + \nabla^2 \Psi) \frac{\partial^2 \Phi}{\partial \pi^2} - \frac{1}{a^2 \cos^2 \phi} \frac{\partial^2 \Psi}{\partial \lambda \partial \pi} \frac{\partial \Phi}{\partial \lambda \partial \pi} - \frac{1}{a^2} \frac{\partial^2 \Psi}{\partial \phi \partial \pi} \frac{\partial^2 \Phi}{\partial \phi \partial \pi} \right] \]  

(3)

The only requirement for obtaining the dynamical fields from the EPV field is a proper inversion algorithm. Recently DE devised such an algorithm, using the non-linear balance equation of Charney (1955) to supply the relation between the \( \Psi \) and \( \Phi \) fields:

\[ \nabla^2 \Phi = \nabla \cdot (f \nabla \Psi) + \frac{2}{a^4 \cos^2 \phi} \frac{\partial}{\partial (\lambda, \phi)} \left( \frac{\partial \Psi}{\partial \lambda}, \frac{\partial \Psi}{\partial \phi} \right), \]  

(4)

the last term being the Jacobian. This non-linear balance condition is capable of handling a flow with large curvature, as long as it remains inertially stable.
The applied boundary conditions are as follows: At the lateral boundaries the observed geopotential on the boundary is used. On the horizontal boundaries the conditions are $\partial \Phi / \partial x = -\theta$ and $\partial \Psi / \partial \pi = -\theta / f$. To perform the EPV inversion, i.e., to solve equations (3) and (4) for the height and streamfunction, we also need the appropriate EPV and $\theta$ fields, along with initial guesses for geopotential height.

From the geopotential height fields, a first-guess streamfunction field is obtained from the quasi-geostrophic assumption:

$$\Psi^0 = \frac{\Phi}{f}. \quad (5)$$

A successive overrelaxation method (SOR) is used to solve the system (3) - (5) (see DE). Note that the equations are not balanced if $q$ is negative, and the method, therefore, can not adequately describe fields in an area of negative EPV.

### 2.3 Model and data

This study was carried out using the HIRLAM model system, described by Gustafsson (1993). The model has a horizontal grid spacing of 0.5 degrees in a rotated Gaussian grid, corresponding to a resolution of about 55 km. In the vertical there are 16 levels, determined by a hybrid vertical coordinate. The input data used in this investigation are derived from the European Centre for Medium Range Weather Forecasts (ECMWF) uninitialized analyses, available every six hours. The analyses of potential temperature and geopotential height given at the mandatory levels were then used to compute EPV on pressure surfaces according to the centered finite-difference analogue of equation (3). EPV is obtained at levels midway between every two mandatory levels, ranging from 950 to 125 hPa. Potential temperature at 950 hPa (1000–900 hPa average) and at 125 hPa (150–100 hPa average) specifies the lower and upper boundary conditions, respectively.

### 2.4 PV and $\theta$ anomalies

A variant of the piecewise EPV inversion method in DE is used to quantify the interactions between isolated portions of the fluid. The procedure requires defining a time mean state, the perturbation fields and their associated EPV anomalies, as well as a suitable quantity which can be used to measure the contribution of various processes. We define a time mean state EPV field as $\bar{q}$ and a perturbation EPV field as $q'$; such that

$$q_{tot} = q(\lambda, \phi, \pi, t) = \bar{q}(\lambda, \phi, \pi) + q'(\lambda, \phi, \pi, t) \quad (6)$$

and do the same for $\theta$:

$$\theta_{tot} = \theta(\lambda, \phi, \pi, t) = \bar{\theta}(\lambda, \phi, \pi) + \theta'(\lambda, \phi, \pi, t), \quad (7)$$

The mean state is defined as either the time-average over the 42 hour period 00 UTC 2 February – 18 UTC 3 February 1991 or the time-average over the 24 hour period 12 UTC 2 February – 12 UTC 3 February 1991 (see below). The suitability of the time mean for defining PV and $\theta$ anomalies has been demonstrated by DE.

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1These levels are: 1000, 900, 800, 700, 600, 500, 400, 300, 250, 200, 150, 100 hPa.
An anomaly \((q_i)\) is a part of the perturbation field denoted by \(q'\) in equation (6) through:

\[
q' = \sum_{i=1}^{n} q_i.
\]

where \(n\) stands for the total number of anomalies in the perturbation field. We shall now describe in some detail how the piecewise inversion was performed.

Isolation of an anomaly of interest is carried out by selecting as the anomaly the grid points within a closed 0-contour in the perturbation field, \(q'\). In some cases the 0-contour turned out to surround more than one anomaly. We therefore selected a rectangular area containing only the anomaly of interest. The anomaly was then defined as the set of grid points within the closed boundary of the 0-contour that lies within the rectangular area. The same applies to the \(\theta\) anomalies at the upper and lower boundaries. In this section we will write a few descriptive relations between the fields and normally only write the relations for the \(q\) fields, but similar relations hold for \(\theta\) as well. Note that in the case of \(\theta\), we are only concerned with anomalies at the upper and lower boundaries, since those anomalies are equivalent to EPV anomalies, as explained by, e.g., Hoskins et al. (1985).

We now, following DE, invert each EPV field. This is carried out in two steps: First we invert the full EPV field, \(q_{tot}\), and then we invert the EPV fields which were obtained by subtracting a given anomaly from the total EPV field, \((q_{tot} - q_i)\), for \(i = 1, \ldots, N\), where \(N\) is the number of selected anomalies, \(N\) being equal to 3 in this case (see subsection 3.1). This results in the corresponding geopotential height fields, denoted by \(\Phi_{tot}\) and \((\Phi_{tot} - \Phi_i)\) \(\forall i = 1, \ldots, N\). By subtracting each \((\Phi_{tot} - \Phi_i)\) field from the \(\Phi_{tot}\) field we obtain the geopotential height field that corresponds to the EPV anomaly in question, denoted by \(\Phi_i^{*}\). That is:

\[
\Phi_{tot} - (\Phi_{tot} - \Phi_i) = \Phi_i^{*}.
\]

We now calculate the cumulative effect of the anomalies not explicitly removed, that is the anomalies with \(i = N + 1, \ldots, n\). This residual field can be written as:

\[
q_{res} = \sum_{i=N+1}^{n} q_i.
\]

The calculation is carried out in a manner analogous to that for the isolated anomalies. We first insert the average field into the total field, \(q_{tot}\), everywhere except where we have previously done so. This gives a field denoted by \((q_{tot} - q_{res})\). We now invert this field as we did the others and get the corresponding geopotential height field, \((\Phi_{tot} - \Phi_{res})\). Then we subtract this field from the total geopotential height:

\[
\Phi_{tot} - (\Phi_{tot} - \Phi_{res}) = \Phi_{res}^{*}.
\]

We now have geopotential height fields corresponding to the total EPV and \(\theta\) fields, the selected EPV and \(\theta\) anomalies, and the residual EPV and \(\theta\) perturbation fields.

Davis (1992) explored different methods to perform piecewise inversion. The method described by equation (9) was termed ST (subtraction from total) by Davis. Following Davis (1992) we also use another method termed AM (addition to the mean). The reason for this is that according to Davis (1992) the average of the resulting fields from these two methods gives results very similar to a linearized method used by DE, which Davis termed FL (full
linear). This linearization analogy implies that we can, to a good approximation, add the geopotential fields corresponding to the anomalies, residual and average EPV and $\theta$ fields, to get the total geopotential field. Clearly, the same property would apply to QGPV, but Ertel's PV is much more general, and hence, more accurate than the quasi-geostrophic counterpart.

To proceed with AM, we first calculate the EPV and $\theta$ anomaly fields by subtracting the $(q_{\text{tot}} - q_i)$ field from $q_{\text{tot}}$ for all $i = 1, \ldots, N$ of interest. We add the EPV anomalies of interest to the average field. That is, we calculate the fields $(\bar{q} + q_i)$ for all $i = 1, \ldots, N$. Then we add the full perturbation field excluding the selected anomalies to the average to get the field $(\bar{q} + q_{\text{res}})$. We now have one field for each of the selected anomalies, and furthermore, the residual fields for EPV and $\theta$.

We invert these fields non-linearly to get the corresponding geopotential height fields. Each EPV anomaly is inverted along with the average $\theta$ field, and each $\theta_i$ anomaly field is inverted along with the average EPV field. This results in the geopotential height fields, denoted as $(\Phi_{\text{mean}} + \Phi_i)$, $\forall i = 1, \ldots, N$. Similarly, the fields corresponding to the EPV and $\theta$ residuals are inverted, giving a residual geopotential height field, denoted $(\Phi_{\text{mean}} + \Phi_{\text{res}})$. We now subtract the balanced mean fields (inverted mean EPV and $\theta$ fields) from these geopotential height fields to get the geopotential height fields corresponding to the anomalies and the residual:

$$(\Phi_{\text{mean}} + \Phi_i) - \Phi_{\text{mean}} = \Phi_i^*$$

and

$$(\Phi_{\text{mean}} + \Phi_{\text{res}}) - \Phi_{\text{mean}} = \Phi_{\text{res}}^*,$$

where $i = 1, \ldots, N$, while $\Phi_{\text{res}}^*$ contains the remainder of the perturbation field, in accordance with (10).

We now take the average of the solutions calculated with the two methods to get:

$$\Phi_i = \frac{1}{2}(\Phi_i^* + \Phi_i^{**}),$$

$$\Phi_{\text{res}} = \frac{1}{2}(\Phi_{\text{res}}^* + \Phi_{\text{res}}^{**}).$$

Finally, we evaluate how well the different terms contributing to the total geopotential field add up. This is done by computing "an error term", $\Delta \Phi$, which is defined by the equation:

$$\Delta \Phi = \Phi_{\text{tot}} - \Phi_{\text{mean}} - \sum_{i=1}^{N} \Phi_i - \Phi_{\text{res}}.$$

In principle, the smallness of $\Delta \Phi$ is not ensured since the inversion methods we have used are non-linear. However, due to the fact that we have combined methods ST and AM in a manner which closely resembles the linearized method FL, we can expect the term $\Delta \Phi$ to be small. In the inversions performed in this study, the very largest $\Delta \Phi$ we found was 150 m$^2$ s$^{-2}$, corresponding to a geopotential height error of 15 m, while the average absolute value of the height error was 6.3 m (see "Error" in Tables 1-5). Based on this small error we conclude that we can confidently compare the contributions from the different EPV anomalies to the total flow, based on the results from the inversion calculations.

To analyze the geopotential height field corresponding to a particular EPV anomaly we locate the position of the low center in each of the N anomaly fields. If the value is negative, then the geopotential height is higher without the corresponding anomaly than with it, which
in turn means that the low is less deep. Negative values therefore show how much deeper a low is with a particular anomaly in the perturbation field than without. This gives a measure of the importance of a particular anomaly in the deepening of the low. The results from this analysis for the three anomalies considered can be seen in Tables 1–5. Also shown are geopotential heights associated with $\Phi_{\text{tot}}$, $\Phi_{\text{mean}}$, $\Phi_{\text{res}}$ and $\Delta\Phi$.

3 Piecewise EPV inversions applied to the 2–3 February 1991 cyclone

3.1 EPV partitioning

The findings of KT2 suggested that three significant and dynamically distinct anomalies could be identified: A positive EPV anomaly propagating along the tropopause, an anomaly associated with the surface baroclinicity, and the positive lower-tropospheric EPV anomaly produced mostly by latent heat release due to condensation. To elaborate on these results we have investigated the following three EPV anomalies: a) surface $\theta'$, a 950 hPa potential temperature anomaly. A positive surface $\theta'$ is equivalent to a positive EPV anomaly (Hoskins et al. 1985). Surface $\theta'$ is normally created by warm and cold advection in the lower troposphere during baroclinic wave growth. Surface fluxes of heat can also contribute to this quantity. b) LPV, a positive low-level EPV anomaly (below 500 hPa). In the real atmosphere, latent heat release leads to the formation of positive EPV anomalies at low levels and negative EPV anomalies at upper levels. In a dry atmosphere, LPV can be created only if there is a lower tropospheric gradient of EPV. c) UPV, a positive upper-level EPV anomaly (above 500 hPa). This anomaly is typically related to a lowering of the tropopause in a cold air mass.

Having performed the inversions we shall focus on the impact of the anomalies on the 900 hPa height field. The reason for choosing the 900 hPa level is that this is the lowest model level above the boundary, see subsection 2.3. Figs. 1a-c show the analyzed height field at 900 hPa at 18 hour intervals, together with 900 hPa potential temperature and upper level potential vorticity. In (a) the low-level cyclogenesis is just about to start near the lower left hand corner of the figure. As the cyclone starts to deepen (b), we see strong temperature advection at low levels, while the upper level potential vorticity anomaly starts approaching the cyclone, as discussed by Kristjánsson and Thorsteinsson (1994). The "merging" of the two systems, suggested by Fig. 1c, will now be investigated further.

3.2 EPV inversions

Referring to (6), we split the flow into a mean flow and a perturbation from the mean. The mean is defined as the average of the 6-hourly analyses over the 42 hour period beginning 00 UTC 2 February. The mean 400 hPa EPV, 900 hPa EPV and 950 hPa potential temperature used for the inversion procedure are shown in Figs. 2a, b and c, respectively. Assuming (see Hoskins and Berrisford, 1988) that the tropopause can be defined as a surface with $\text{EPV}=20$ dPVU (1 dPVU = 0.1 PVU), Fig. 2a indicates an intrusion of stratospheric air at 400 hPa over a fairly large region west of the cyclone track, shown in Fig. 3. We suggest this is a result of tropopause folding, and hence is a manifestation of the baroclinic energy conversion process associated with this explosive cyclone. The question therefore arises as to what extent the UPV signature can be regarded as a cause of cyclone deepening as opposed to being a
Figure 1: Analyzed 900 hPa height field (solid lines) and potential temperature field (dashed lines) at 900 hPa at: a) 00 UTC 2 February 1991; b) 18 UTC 2 February 1991; c) 12 UTC 3 February 1991. The shaded areas correspond to $4 \text{ PVU} \geq \text{ EPV} \geq 2 \text{ PVU}$ on 400 hPa.
Figure 2: Forty-two hour mean: a) EPV (in dPVU) at 400 hPa; b) EPV (in dPVU) at 900 hPa; c) lower boundary $\bar{\theta}$ (K) at 950 hPa. Twenty-four hour mean: d) EPV (in dPVU) at 400 hPa; e) EPV (in dPVU) at 900 hPa; f) lower boundary $\bar{\theta}$ (K) at 950 hPa. (1 dPVU = 0.1 PVU)
result of the cyclone deepening. We will return to this question as we look into the time evolution of the potential vorticity in subsection 3.3.2.

Figure 3: The upper level (x, 6 hr. position) and surface cyclone (o) track and the difference in geopotential height (in m) between original and balanced \( \Phi \)-fields at 900 hPa at 18 UTC 2 February 1991.

In performing the inversion computations described above on the original grid, certain convergence problems were encountered. To overcome these convergence problems, as well as speeding up the convergence, the grid resolution is decreased by a factor of 4 in both x and y directions, from 55 km to 220 km. The EPV inversion is first performed on this coarse grid. Subsequently a local bi-cubic interpolation is used to interpolate the coarse grid height and streamfunction to a resolution of 110 km for use as the first guess in a second inversion step performed on this finer grid. Finally, a local bi-cubic interpolation is again used to interpolate to a horizontal resolution of 55 km. These are our presented values for the height and the streamfunction. Attempts to perform the inversion once more at this level resulted predictably in similar convergence problems as originally encountered. The reason for these convergence problems is not fully clear but the difficulties tend to start within areas of negative potential vorticity, where, as mentioned in subsection 2.2, the balance requirements needed for a proper inversion are not satisfied. The intense cold advection to the south and southwest of Greenland indicated in Fig. 1a, accompanied by heating from the warm ocean surface below, leads to negative EPV-values (through the \( \partial \theta / \partial \pi \) term in (3)) over a large region south of Greenland, Fig. 2b. Fig. 2c shows that there is a strong gradient in potential temperature between the cold air over and to the east of Labrador and the warm air over the eastern part of the North Atlantic. This warm air is advected northward by the cyclone to the effect that the 278 K isentrope extends from a latitude of 40°N at 50°W to a latitude of 72°N at 10°E. Figs. 2d, e and f show results analogous to those in Figs. 2a, b and c, respectively, except that the averaging is now done over the 24 hour period beginning 12 UTC 2 February 1991. Qualitatively, the main features are the same as in Figs. 2a, b and c, the largest difference being found in the mean 400 hPa EPV, which is on average higher by 0.5 PVU for the 24 hour average than
for the 42 hour average. We also find deviations in $\bar{\theta}$ at 950 hPa where the lower left corner is considerably warmer for the 42 hour mean than for the 24 hour mean. The reason for showing the 24 hour averages is that those will be used in conjunction with the sensitivity runs, described in subsection 3.3.2.

### 3.3 Results from the EPV inversions

#### 3.3.1 A comparison between analyzed and balanced fields

In Fig. 3 we show the difference between the balanced solution of the full field and the actual heights of the same field for 18 UTC 2 February. As seen we find rather small differences, usually much less than 40 gpm at 900 hPa. The exception is the statically unstable region discussed above, where considerably larger discrepancies were found. Fortunately that area is far enough away from the center of the cyclone that it should not affect the low-level circulation associated with it.

As another measure of how well our balanced fields match the original ones, we show in Fig. 4 a comparison between analyzed (left column) and balanced (right column) nondivergent winds in, respectively, the upper (top row) and lower (bottom row) troposphere. The wind direction is generally almost the same with the exception of the area near the southern tip of Greenland, where there were large differences in geopotential height due to negative EPV, see Fig. 3. Differences in wind speed between analyzed and balanced nondivergent fields are generally smaller than 10 m s$^{-1}$.

#### 3.3.2 Evolution of the EPV anomalies

To facilitate the understanding of the various aspects of cyclone dynamics dealt with here, we shall accompany analysis fields with results from different model simulations, as in the previous studies, KT1 and KT2. As these model runs were explained in detail in KT2, only a brief definition will be given here: CONTROL = Simulation starting at 12 UTC 2 February 1991, using ECMWF analysis at the boundaries; NOLAT = As CONTROL, except that latent heat of condensation is omitted in the thermodynamic energy equation; RUN00 = As CONTROL, except starting at 00 UTC 2 February 1991.

Figs. 5a-c show that the EPV field above the surface low is characterized by a steadily lowering tropopause, low-level potential vorticity generation, and an apparent merging between these two EPV maxima at 12 UTC 3 February. A question we wish to address (see first paragraph section 3.2) is to what extent this merging is simply the manifestation of the occluding process, as opposed to signifying constructive interference by two independent phenomena, i.e., the upper level and the lower level anomalies.

Shown in Fig. 6 is the evolution of the 400 and 900 hPa EPV perturbation fields and the $\theta'$ perturbation field, as well as relative humidity. Figs. 6a-c show the dry 400 hPa EPV anomaly of interest (the UPV anomaly) as a rather well defined zone of $\text{EPV} \approx 2.0$ PVU or more, moving ENE originally and later NNE. At the final time shown it is almost vertically aligned with the surface cyclone. The 900 hPa EPV anomaly (Figs. 6d-f) originates from latent heat release, as will be demonstrated below (NOLAT); see also KT1. It has a very different evolution from that of the 400 hPa anomaly, being small at the initial time (Fig. 6d), but then increasing in the developing phase of the cyclone (Fig. 6e) and having a magnitude of more than 1 PVU at the final time (Fig. 6f). Note also how the low-level anomaly coincides with the area of large relative humidities, as is required for latent heat release to take place.
Figure 4: a) Winds at 400 hPa, 18 UTC 2 February 1991 from model analyzed fields; b) As a), but showing balanced winds based on inverted total EPV-field; c) Winds at 900 hPa, 18 UTC 2 February 1991 from model analyzed fields; d) As c), but showing balanced winds based on inverted total EPV-field.
Figure 5: EPV cross sections from CONTROL: a) at 00 UTC 2 February 1991; b) at 18 UTC 2 February 1991; c) at 12 UTC 3 February 1991. The line PVU=1.5 has been enhanced for clarity. (Locations of cross sections are shown in Fig. 2 a.)
Figure 6: Perturbation fields associated with 400 hPa EPV (in dPVU, top row), 900 hPa EPV (in dPVU, middle row) and lower boundary $\theta$ (in K, bottom row). The shaded areas in the top and middle row correspond to relative humidity $\geq 90\%$ at 400 and 900 hPa, respectively.
The $\theta'$ perturbation, shown in Figs. 6g-i, mainly consists of a positive anomaly, located just on the warm side of the surface low and a negative anomaly behind the cold front to the west and northwest of the surface low. The positive anomaly reaches its largest magnitude at the initial time, exceeding 8 K, Fig. 6g. At 18 UTC 2 February (Fig. 6h) and 12 UTC 3 February (Fig. 6i) it is located in the warm sector of the cyclone and with maximum amplitudes of about 5 K and 4 K, respectively. The areal extent of this anomaly decreases markedly during the cyclone evolution, while that of the cold anomaly increases, as the warm air rises and cold air is brought down, hence converting available potential energy to kinetic energy of the disturbance.

3.3.3 Contributions to cyclone deepening

We shall now look at the contributions to the 900 hPa geopotential height fields from the three main EPV anomalies. We first focus on results using model analyses. First, in Figs. 7a-c we see the contribution from UPV at three different times in the storm evolution. Figs. 7d-f show the corresponding result for LPV, while Figs. 7g-i display the contribution from surface $\theta'$. In addition to the change with time of the relative strength of the three anomalies, which will be discussed in more detail below, we also see interesting phase differences: The UPV signature is consistently farther upstream than the other two, although somewhat less so as time progresses. Conversely, the surface $\theta'$ anomaly tends to deepen the low in the direction of the warm air on its downstream flank. The LPV contribution is most closely in phase with the actual surface low, see Fig. 1. Figs. 7j-l show the contribution from the residual field at 900 hPa. The residual field contributes significantly to the weakening of the low over the first 24 hours or so, but from then on it weakens and its effect near the low center is rather small at the final time, see Table 1. By then its impact is largest to the rear of the surface cyclone (Fig. 7l), which corresponds well with the filling given by negative surface $\theta'$ (Fig. 6i). Similarly, comparing Figs. 6g and 7j and Figs. 6b, 6h and 7k, we find that the filling created by negative values of UPV and surface $\theta'$ in the cold air is largely contained in the residual term.

In Fig. 8 we show the sum of all the terms given in each of the three columns in Fig. 7. This corresponds to the deviation from the mean geopotential field, since

$$\sum_{i=1}^{N} \Phi_{i} + \Phi_{res} = \Phi_{tot} - \Phi_{mean}$$  

Hence, Fig. 8 shows the development of the cyclone even more clearly than Fig. 1. At 00 UTC 2 February there is merely a trough at 42°N, 46°W. Eighteen hours later, Fig. 8b, the low center is quite distinct at 50°N, 28°W, and finally at 12 UTC 3 February the cyclone has reached maturity just west of Iceland, and a distinct trough associated with the cold front extends southwards to the west of Ireland.

Due to the choice of only three EPV-anomalies, all of which were positive, we obtained in Fig. 7 a fairly large residual term, representing mostly anticyclonic circulation. How does this affect the interpretation of the height fields from the three PV-anomalies? A partial answer to this question is given by Fig. 9, which shows the sum of the geopotential heights from, respectively, the residual field and the upper-level PV-anomaly at the time when we expect the upper-level PV-anomaly to be most important. We see then, that despite the large positive values associated with the residual term, there still remains a strong cyclonic circulation, when the residual height field is added to the UPV-height field. The geopotential
Figure 7: Contributions to perturbation heights (in gpm) at 900 hPa from upper EPV (q at 500 to 250 hPa, top row), lower EPV (q at 900 to 600 hPa, upper middle row), lower boundary θ (lower middle row), and the residual field (bottom row). The position of the surface low center is denoted by L.
Figure 8: Perturbation (total - mean) geopotential height field at 900 hPa from analyses at:
a) 00 UTC 2 February; b) 18 UTC 2 February; c) 12 UTC 3 February.
height of this cyclone is about -12.5 m, its position being shifted slightly to the southeast of the analyzed cyclone, Fig. 1. We may thus conclude that the UPV-anomaly is large enough not to be obscured by the negative EPV-anomaly associated with the residual term.

![Figure 9](image_url)

Figure 9: The sum of geopotential heights at 900 hPa associated with, respectively, the residual term and the upper-level potential vorticity anomaly at 12 UTC 3 February 1991.

Tables 1 - 5 summarize the contributions to 900 hPa height at the cyclone center for different model runs every 6 hours. Considering first the geopotential heights from the analysis data, Table 1; we see as in Fig. 6 a systematic change in the role of the three anomalies as time evolves. The surface $\theta'$ anomaly gives by far the largest contribution of the three at the initial time, and remains very significant over the next 12 hours. Over the final 24 hours, however, the surface $\theta'$ anomaly weakens and its role is gradually overwhelmed by the contributions from the other two anomalies, UPV and LPV. The LPV anomaly is very weak at 00 and 06 UTC 2 February. At 12 UTC its contribution has become as large as that of the other two anomalies, and after that it continues to increase, except during the last 6 hours, when the cyclone has reached maturity. The contribution of the upper level anomaly is already quite large at 06 UTC 2 February. From then on it grows quite slowly apart from the final 6 hours when it grows rapidly, reaching its highest value at 12 UTC 3 February.

Table 2 shows results analogous to those in Table 1, except that the averaging is now done over the final 24 hours only. Qualitatively, the main features are the same as in Table 1, the largest difference being found in the contribution from UPV, which is on average weaker by 17 gpm for the 24 hour average than for the 42 hour average.

We shall now compare Table 2 to sensitivity experiments with the HIRLAM model, using the analysis at 12 UTC 2 February as initial data. Table 3 shows results from the CONTROL simulation. They show a large similarity to those of the analyses, hence justifying the use of results from CONTROL in the assessment of the cyclone development, as done in KT2. The most significant discrepancy is a slightly too weak contribution from UPV, while minor discrepancies associated with LPV and surface $\theta'$ have varying signs.
<table>
<thead>
<tr>
<th>Date/Time</th>
<th>2/06</th>
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<td>918</td>
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<td>765</td>
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Table 1: Contributions to geopotential height (in m) at 900 hPa at the location of the surface cyclone, from different anomalies at 6 hour intervals, based on model analyses. A 42 hour average from 2/00 to 3/18 was used.

<table>
<thead>
<tr>
<th>Date/Time</th>
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<tr>
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<tr>
<td>Error</td>
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<td>-1</td>
<td>-6</td>
<td>7</td>
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<td>890</td>
<td>821</td>
<td>750</td>
<td>723</td>
</tr>
</tbody>
</table>

Table 2: As 1, except that a 24 hour average from 2/12 to 3/12 was used.

In KT2 it was shown that omitting latent heat release led to a significantly weaker simulated cyclone. This result can be understood if we compare Table 4 to Table 3. Already after 6 hours, i.e., at 18 UTC 2 February, the height associated with LPV is 69 gpm higher in NOLAT than in CONTROL. Six hours later the difference has increased to 117 gpm. At the same time the height associated with UPV is now 27 m lower than in CONTROL. This trend continues, and after 24 hours, at 12 UTC 3 February, the differences corresponding to LPV and UPV are +143 gpm and -57 gpm, respectively. This dipole structure is compatible with the notion that diabatic heating creates positive potential vorticity below the level of heating and negative potential vorticity above the level of heating. At all times the contribution of the surface $\theta'$ anomaly is similar in NOLAT and CONTROL, i.e., displaying a systematic decrease with time. The fact that the height anomaly corresponding to LPV in NOLAT is quite small, apart from the first 6 hours (Table 4), confirms the claim already made that the LPV anomaly is largely a result of latent heat release. This is demonstrated in Fig. 10, which shows the LPV contribution to 900 hPa height fields for the two cases, CONTROL and NOLAT, at +18 hours.

### 3.3.4 Sensitivity to the initial state

Finally, we can investigate the sensitivity to the initial state by comparing the results from RUN00 to those of the other simulations. As explained in KT2, RUN00 had large errors in both positioning and intensity of the cyclone, similar to that of many operational models.
Figure 10: Geopotential height field at 900 hPa associated with the low level EPV anomaly (900 to 600 hPa) at 06UTC 3 February 1991 from different runs: a) CONTROL run; b) NOLAT run.
at the time. Through this comparison we wish to answer the following two questions:

1) Why did the RUN00 simulation fail to deepen and position the cyclone correctly, as opposed to CONTROL?

2) Is the upper level anomaly truly a separate entity that influences the low-level flow, or is it strongly affected by what happens at low levels?

To answer the first question, we first look at the contribution from the surface $\theta'$ anomaly at 12 UTC 2 February, since it was suggested in KT2 (Fig. 10 of that paper) that at this time the low-level contribution was much weaker in RUN00 than in CONTROL. It was further suggested that this was probably mainly due to errors in the analysis at 00 UTC 2 February. Fig. 11 confirms this hypothesis by showing a much stronger cyclone resulting from the surface $\theta'$ anomaly in Fig. 11a than in Fig. 11b. The stronger cyclone will in turn give rise to stronger warm advection ahead of the low and stronger cold air advection to the rear of the low, as seen by assuming approximately geostrophic winds and holding Fig. 11 together with the temperature field in Fig. 2c. Through the omega-equation (Holton, 1992), this gives rise to vertical motions that further intensify the cyclone in CONTROL, as compared to RUN00, due to baroclinic energy conversion.

Comparing the surface $\theta'$ contributions in Tables 3 and 5 we see at 12 and 18 UTC 2 February a large difference, i.e., 58 and 38 gpm respectively, while the difference in the contribution from UPV and LPV at these times is fairly small. This underlines the role of the low-level baroclinicity at the early stages. Six hours later, at 00 UTC 3 February, the largest difference between Tables 3 and 5 is in the LPV contribution, while at the two final times, 06 and 12 UTC 3 February, the largest difference stems from the UPV contribution.
Figure 11: Geopotential height field at 900 hPa associated with the lower boundary $\theta$ anomaly at 12 UTC 2 February: a) CONTROL run; b) RUN00.
This seems to suggest a vertical propagation of an EPV anomaly as the cyclone deepens. Indeed, an estimate of the vertical propagation velocity can be carried out, yielding:

\[
\omega \approx \frac{-500 \text{ hPa}}{18 \text{ h}}
\]

\[
\downarrow
\]

\[
w = -\frac{\omega}{\rho g} \approx 0.08 \text{ m s}^{-1}
\]

<table>
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<td>978</td>
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<td>896</td>
<td>841</td>
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</tbody>
</table>

Table 5: As Table 1, except based on RUN00.

This is indeed a reasonable vertical velocity within the cyclone, suggesting that such a vertical propagation of EPV may have occurred. But, how does this relate to our question (2) above? Fig. 12 compares the contribution from UPV at the final time in three cases; CONTROL, ANA42 and RUN00. First in Fig. 12a we see how much the UPV-anomaly is "in error" in the CONTROL run. This is seen to occur mainly between 50 and 55°N, i.e., well to the rear of the cyclone of interest, whereas the error near Iceland is very small. On the other hand, comparing the contribution from the UPV-anomaly between CONTROL and RUN00, Fig. 12b, we see an underestimation of the cyclone by up to 40 gpm in RUN00. We suggest that this feature is a result of the vertical propagation of the initially low-level EPV anomaly mentioned above. Hence, both the UPV field and its associated geopotential height field are superpositions of two features, the largest one being identical in all the runs, due to its having been advected horizontally, uninfluenced by the low level flow; and a secondary feature being caused by vertical propagation of an EPV anomaly related to the low-level cyclonic development early on. Since this initial development is too weak in RUN00, so is the secondary UPV feature at the final time. But, it should be emphasized that the "main feature" everywhere accounts for at least two thirds of the geopotential field associated with UPV.

4 Discussion

Despite the valuable insight gained in this study there are a number of questions that may be raised regarding the methodology applied. One such issue concerns the selection of the anomalies. Based on our preliminary results in KT2, we started out by subjectively identifying three major positive anomalies that we focused on, i.e., UPV, LPV and surface \(\theta'\). A priori, it is to be expected that these three positive anomalies together overestimate
Figure 12: Geopotential height difference fields at 900 hPa associated with the upper level EPV anomaly at 12 UTC 3 February: a) Difference between CONTROL run and ANA42 analysis; b) Difference between CONTROL run and RUN00.
the cyclone deepening, as positive anomalies induce deepening and negative anomalies induce filling. Hence, a fairly large residual term of opposite sign appears in Tables 1-5, since it contains all the contributions from negative anomalies. The assumption that the cyclone can be viewed as consisting of contributions from only three positive anomalies, two of them being defined as finite differences over several kilometers, is clearly an oversimplification. By explicitly defining and inverting negative EPV anomalies the residual term would have become smaller, rendering interpretation somewhat easier. Nevertheless, we feel that this exercise has rendered valuable information on the dynamics of the cyclone. Clearly, there are other cases in which a different choice of anomalies would be more suitable. Furthermore, in cases with less synoptic guidance it may not be clear how to select the relevant anomalies.

Another related issue is how to define an anomaly, once it has been selected. This issue can be split into two problems, i.e., how to define a mean field and how to select the anomalies from the perturbation field, in terms of grid points. Changing the definition of the mean field would implicitly change the perturbation field, thus resulting in some arbitrariness when defining the anomalies. We have chosen simply to use a time mean for the whole study period. This corresponds approximately to the life cycle of one extratropical cyclone, but a longer period can also be used, as suggested by Davis and Emanuel (1991).

Once a definition of the mean field has been decided upon, the problem of actually selecting the grid points that belong to a specific anomaly arises. Here there is inherent arbitrariness since the selection criteria can be varied (see section 2.4 for our selection criteria). For instance, we have rerun the inversion procedure where the surface \( \theta' \) anomaly had a broader spatial extent than in Figs. 7g-i. This typically resulted in the associated height field being less confined to the location of the surface cyclone, and consequently the residual field became even larger than that shown here.

We have not shown any results concerning the upper boundary \( \theta' \). This is due to our assumption that anomalies there, if any, will not contain any significant meteorological signature, since the upper boundary lies entirely within the stratosphere.

Besides helping to understand explosive cyclone dynamics, EPV inversion may conceivably have applications related to weather forecasting or analysis, see e.g., Mansfield (1994). For instance, if an error in a first-guess is known to be due to a particular feature at either the surface, low levels or upper levels, one way to correct it may be to modify the corresponding EPV field, rather than the pressure field. The advantage of using EPV being that the three-dimensional aspect of the correction would be inherently taken care of.

5 Summary and conclusions

We have investigated an explosive synoptic-scale cyclone in the North Atlantic in the framework of Ertel’s potential vorticity (EPV). Following a method recently developed by Davis and Emanuel (1991) we first identified a few distinct anomalies in the EPV field. By successively inverting parts of the potential vorticity field, we then quantified the contributions from these anomalies to the geopotential field. Despite the non-linearity of EPV, the contributions add up quite well, due to the particular procedure employed, as suggested by Davis (1992). The inversions were performed on both analysis data and on data from different simulations, using the HIRLAM model. Those simulations were previously described by Kristjánsson and Thorsteinsson (1994, 1995), where preliminary, qualitative EPV diagnostics were presented. The results obtained here support the main conclusions from that study and
at the same time put them on a more firm, quantitative footing. Our main findings from this investigation are:

- The EPV inversion methodology of Davis and Emanuel (1991) seems suitable for quantifying the contributions from different sources to the total geopotential field associated with the 2-3 February 1991 explosive cyclone near Iceland.

- The upper level anomaly has been shown to be mainly a separate entity that influences the low-level flow, but to be partly due to vertical propagation of EPV from lower layers.

- The inversion results support the earlier conclusion that insufficient surface baroclinicity early on was probably the main reason why many operational forecasts failed to predict the explosive cyclone deepening.

- There appears to be an upper limit for how fine the horizontal grid resolution can be if the Davis and Emanuel iteration scheme is to converge. The reason for this is not fully clear but the difficulties tend to start within areas of negative potential vorticity. In the present study the resolution limit was at 110 km.

- There is some arbitrariness in specifying and defining the anomalies, but this can be reduced by using more restrictive selection criteria.

6 Acknowledgements

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