Technical Report No 27

HIGH RESOLUTION LIMITED AREA MODELLING

Sources of noise in the "physics"; a preliminary study

A. McDonald, Met Éireann

Norrköping, November 1996

Sources of noise in the 'physics' ; a preliminary study.

A. McDonald, Met Éireann

1. INTRODUCTION

Because the semi-Lagrangian semi-implicit scheme has an extremely liberal stability criterion we can now use very large time steps to integrate the dynamics in primitive equation models. This prompts the following question. Can we use equally large time steps when updating physical processes?

There is an additional reason for worrying about integrating the 'physics' with relatively big time steps even if one does not wish to embrace the semi-Lagrangian approach. The next generation of models is likely to have much thinner layers in the vertical. This fact may also force us to use a smaller time step than is necessary for an accurate integration if we have not carefully chosen our numerical schemes for the physical parameterizations.

In order to use bigger time steps in the 'physics' we must make sure that the time discretization schemes we use are accurate, stable, and non-oscillating. In section 2 some possibly unexpected sources of inaccuracy, instability, and unwanted oscillations, which may arise when large time steps are used in the 'physics', are examined. This is done by employing some widely used strategies to integrate a number of simple differential equations which are typical of the kind used in the HIRLAM physical parameterization schemes. In section 3 an example of a spurious oscillation which arose in an integration of the HIRLAM model is shown and some attempts to control it are described. In section 4 there are some conclusions and suggestions.

2. SIMPLE EXAMPLES OF DANGERS.

2a. Stiffness.

Consider the equation

$$\frac{du(t)}{dt} = -K[u(t) - F(t)] + \frac{dF(t)}{dt}; \quad u(0) = 0. \tag{2.1}$$

K is a constant. The solution is given by

$$u(t) = [u(0) - F(0)]exp(-Kt) + F(t) \tag{2.2}$$

If we choose $K = 100hr^{-1}$ and $F(t) = t/10 + 0.5$, ('t' is in units of hours), then we see that the solution consists af a slowly varying part, F(t), and a rapidly varying part, exp(-100t); see the line made of long dashes in fig. 1. If we are only interested in the slowly evolving part of this solution then we would expect to be able to use a large time step to get as accurate an integration of Eq. (2.1) as we require. Looking naively at fig. 1, a time step of 0.5hr would seem to be a conservative choice. Let us examine what happens when we use this time step with three simple integration schemes, all of which are used widely in the 'physics'. These are the Euler explicit, the trapezoidal, and the Euler implicit schemes. These correspond to $\gamma = 0$, $\gamma = 0.5$, and $\gamma = 1$, respectively, in equation (2.3):

$$(u^{n+1} - u^n)/\Delta t = -K[\gamma u^{n+1} + (1-\gamma)u^n] + KF^n + (F')^n \tag{2.3}$$

With $\Delta t = 0.5$ hr the Euler explicit scheme is unstable. The values of u for the first five time steps are 0, 25, -1195, 58589, -2870871. This growing oscillation continues until we get a floating overflow. We must use a very small time step ($\Delta t = 0.01hr$) to get a stable and oscillation-free integration.

This illustrates the phenomenon of *stiffness*; the time step is restricted by stability requirements, rather than accuracy. A good working definition of stiffness is as follows. A system is *stiff* if the solution we seek is slowly varying but other solutions exist which decay rapidly.

A stability analysis of the homogeneous part of Eq. (2.3) can be performed by substituting $u(n\Delta t) = \lambda^n \hat{u}$, giving

$$\lambda = \frac{1 + \Delta t K(\gamma - 1)}{1 + \Delta t K \gamma}. \tag{2.4}$$

For the Euler explicit scheme ($\gamma = 0$) we must have $K\Delta t \leq 2$ for stability.

Notice that the trapezoidal scheme ($\gamma = 0.5$) has $|\lambda| \leq 1$. Thus an integration with $\Delta t = 0.5$ hr will be unconditionally stable. This is in fact true, as can be seen from fig. 1, where the solution is displayed as a continuous line. Unfortunately it oscillates about the correct solution before settling down. Stiffness strikes again!

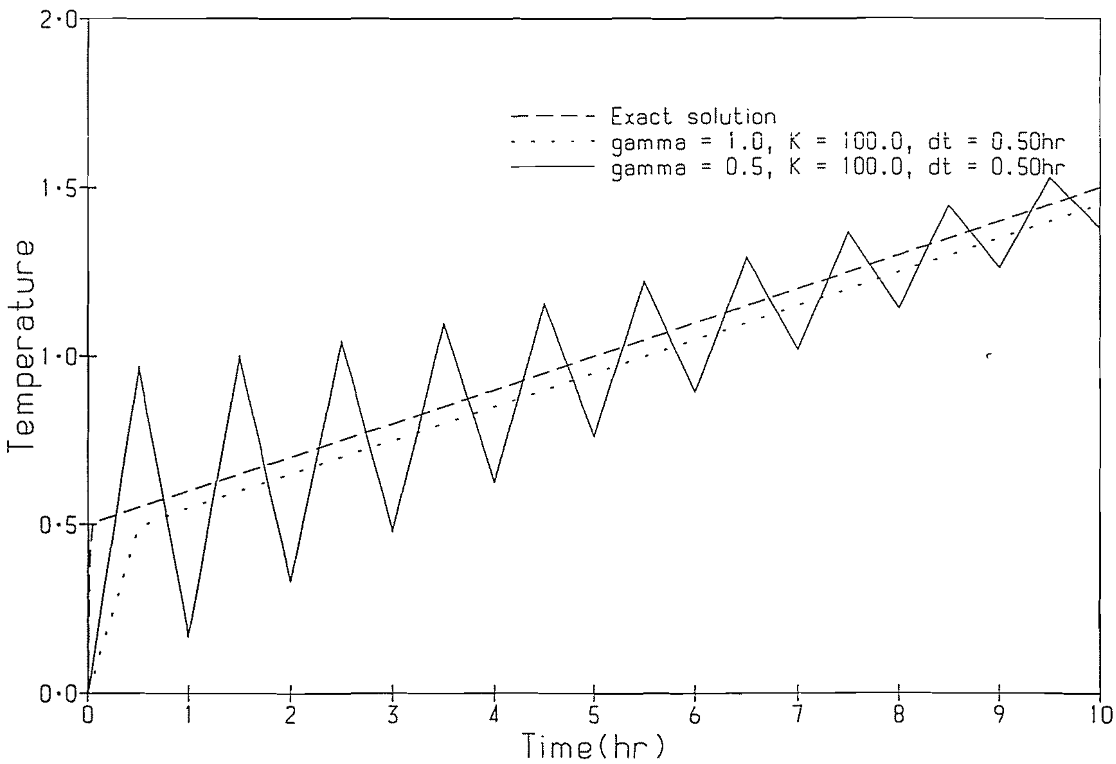

FIGURE 1. Graphs of the solutions to Eq. (2.3) using $\Delta t = 0.5, K = 100.0$ *and various values of* γ.

We can understand what is happening here by noticing that for $\gamma = 0.5$ and $K\Delta t = 50$, $\lambda = -0.92$. Therefore, the homogeneous solution will oscillate, because $u(\Delta t) = -0.92\hat{u}$; $u(2\Delta t) = (-0.92)^2\hat{u} = 0.85\hat{u}$; $u(3\Delta t) = (-0.92)^3\hat{u} = -0.79\hat{u}$, and so on.

Obviously, if we wish to eliminate this oscillation from the homogeneous solution we must choose γ such that $\lambda \geq 0$, that is, $\gamma \geq 1$. With $\gamma = 1$, (the Euler implicit scheme) and $\Delta t = 0.5hr$, then $\lambda = 0.02$ and the transient solution disappears rapidly, as it should. The integration is then stable and non-oscillatory. See the line made with short dashes in fig. 1.

The above simple analysis is actually indicative of some very general results. If I understand the mathematical theorems correctly, then the following is true for stiff systems. 1. No explicit scheme will be stable for a stiff system. 2. The choice of implicit schemes which are stable and non-oscillatory is extremely restricted. 3. $O(\Delta t^2)$ accurate schemes will be computationally expensive. 1 and 2 above, when expressed in precise mathematical terms, are known as the 'second Dalquist barrier'; see Dalquist (1963), and section 6.6 of Lambert (1991). The latter also contains an excellent discussion on the nature of stiffness in sections 6.1 and 6.2.

Where can stiffness arise in the 'physics'? One possibility is if a process is 'switched on' or 'switched off' instantaneously. Since this represents a very fast process indeed it is a prime candidate for stiffness.

2b. **Boundaries.**

Boundary conditions can also be a source of oscillating solutions when large time steps are used. To illustrate this let us solve solve the diffusion equation,

$$\frac{\partial X(z,t)}{\partial t} = \kappa \frac{\partial^2 X(z,t)}{\partial z^2}, \tag{2.5}$$

with the following boundary conditions: $X(0,t) = 0$; $X(Z,t) = 0$; and the following initial condition: $X(z,0) = X_0$ for $0 < z < Z$. The parameter κ is a constant.

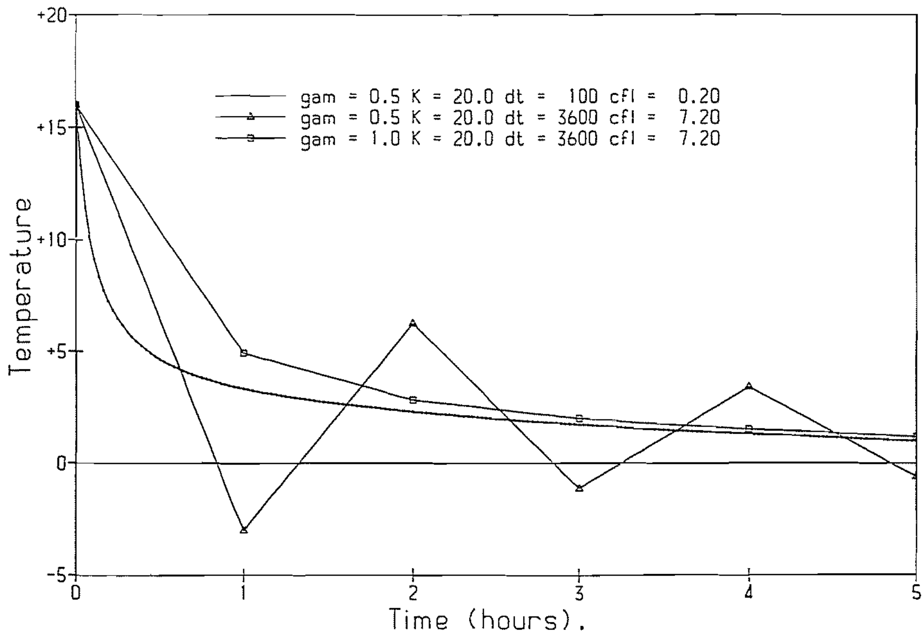

FIGURE 2. Graph of the temperature at $z = 100m$ predicted by Eq. (2.6) for various values of Δt and γ. In the labels within the figure cfl $= \kappa\Delta t/\Delta z^2$.

This is the classic problem of a bar initially at temperature X_0 whose ends are placed against blocks of ice. Physically, the bar gradually cools, most rapidly near the ends, and more slowly at the centre. There is an analytical solution: see Churchill (1963). For our purposes we can also regard it as column model of a tongue of warm air between the cold ground and the cold free atmosphere. Let us discretize as follows:

$$\frac{(X^{n+1} - X^n)}{\Delta t} = \kappa\left[\gamma\left(\frac{\partial^2 X}{\partial z^2}\right)^{n+1} + (1-\gamma)\left(\frac{\partial^2 X}{\partial z^2}\right)^{n}\right] \tag{2.6}$$

$$\left(\frac{\partial^2 X}{\partial z^2}\right)_k = \frac{X_{k+1} - 2X_k + X_{k-1}}{\Delta z^2} \tag{2.7}$$

I have chosen $X_0 = 16°$, $Z = 1600m$, $\kappa = 20sec^{-1}$, and $\Delta z = 100m$. With these choices $\kappa\Delta t/\Delta z^2$ varies between 0.2 and 7.2 as Δt goes from 100sec to 1 hour. Let us examine what happens if we use the trapezoidal scheme to integrate. Since this has $\gamma = 0.5$ it is $O(\Delta t^2)$ accurate, and would seem to be the optimal choice. For small values of $\kappa\Delta t/\Delta z^2$ we get stable and accurate integrations. See the line joining the small dots in fig. 2. This shows the evolution of the temperature at $z = 100m$ over the first five hours of the integration, using a time step of 100 sec.

If we are interested only in the longer-term changes in the temperature at 100m we would like to be able to integrate with a large time step. However, with the trapezoidal scheme a nasty oscillation occurs if we do this. See the line joining the triangles in fig. 2. For this integration $\Delta t = 3600sec$ ($\kappa\Delta t/\Delta z^2 = 7.2$). Notice that the integration is settling down as time goes on; the scheme is 'stable but oscillating'.

The Euler implicit scheme ($\gamma = 1$) has no oscillations. See the line joining the squares in fig. 2., which again shows the integration with $\kappa\Delta t/\Delta z^2 = 7.2$. Of course, there is some loss in accuracy initially.

Sudden jumps in space or in time can lead to unstable or oscillating solutions and must be treated carefully.

2c. Non-linear terms.

The results and arguments of Kalnay and Kanamitsu (1988) are used extensively in this section.

In order to illustrate additional problems posed by 'non-linear terms' when updating the 'physics' with large time steps consider the following simple damping equation.

$$\frac{\partial X}{\partial t} = -KX^{P+1}(t) + S(t) \tag{2.8}$$

To give a meteorological flavour to the interpretation of the results let X be the temperature difference between the ground and air, KX^P represents the exchange coefficient, and all slowly varying processes are included in S, to which we will assign a diurnal cycle: $S = 1 - sin(2\pi n\Delta t/24)$, ($\Delta t$ is measured in units of hours in this section) $K = 10hr^{-1}$ and $P = 3$. With these choices, the solution is slowly varying, see the line joining the dots in fig. 3, and $\Delta t = 0.5$ hr would seem to be sufficient to maintain our required accuracy.

Let us examine first of all a most disconcerting result. A predictor-corrector discretization which treats the linear term implicitly is

$$(\tilde{X} - X^n)/\Delta t = -K(X^n)^P\tilde{X} + S \tag{2.9a}$$

$$(X^{n+1} - \tilde{X})/\Delta t = -K\tilde{X}^P X^{n+1} + S \tag{2.9b}$$

With $\Delta t = 0.5$ hr time step this gives a solution which is stable and non-oscillatory, and *completely wrong*. See the line joining the '+'s in fig. 3, and compare with the correct solution. (An accurate integration can be restored by reducing the time step to 0.166 hr).

The Euler implicit scheme served us well in sections 2a and 2b. Thus, let us try the following scheme with $\gamma = 1$

$$(X^{n+1} - X^n)/\Delta t = -K(X^n)^P[\gamma X^{n+1} + (1-\gamma)X^n] + S^n \tag{2.10}$$

With a $\Delta t = 1.0$ hr time step this produces an unpleasant result. The solution oscillates wildly. See the line joining the squares in fig. 3.

The linear analysis of Kalnay and Kanamitsu shows us that the amplification factor for the scheme of Eq.(2.10) is

$$\lambda = \frac{1 - \alpha(P + 1 - \gamma)}{1 + \alpha\gamma} \tag{2.11}$$

where $\alpha = KX_0^P\Delta t$, X_0 being the equilibrium temperature used to linearise the equation. Thus, the linear stability requirement is violated for large α when $P = 3$ and $\gamma = 1$, since $\lambda \approx -3$. This analysis also tells us that $\gamma = P + 1$ gives a non-oscillating unconditionally stable scheme since $0 \leq \lambda \leq 1$. Such a solution is plotted as the line joining the 'x's in fig. 3.

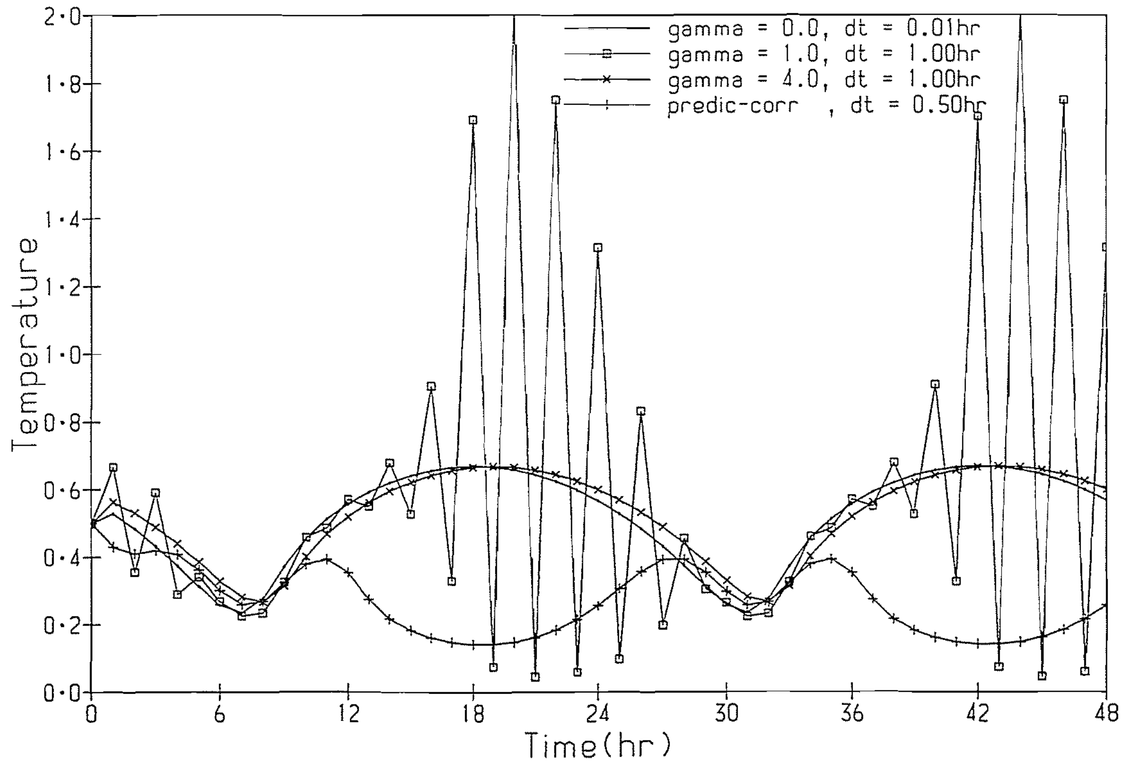

FIGURE 3. Graphs of solutions to Eq. (2.8) using the predictor-corrector scheme, Eq. (2.9) and simple implicit schemes (Eq. (2.10) with various values of γ)

In the HIRLAM model we use three approaches when solving equations containing non-linear terms. (a) Eulerian explicit. We may have to think again about using this scheme for large time steps. (b) Various first order accurate schemes in which we treat the linear terms with varying degrees of implicitness. For instance, we integrate the vertical diffusion scheme as described in Eq. (2.6) with $\gamma = 1.5$. We may wish to ask whether this is sufficiently implicit. (c) Consider the generic equation $dX/dt = f(X)$. In some of the physical parameterization schemes we discretize as follows:

$$\frac{(X^{n+1} - X^n)}{\Delta t} = f(X^n) + \frac{(X^{n+1} - X^n)}{2}\frac{df}{dX}(X^n). \tag{2.12}$$

Notice that if we apply this idea to Eq. (2.8) we get Eq. (2.10) with $\gamma = P + 1$, an unconditionally stable scheme according to the analysis of Kalnay and Kanamitsu, as we discussed in the previous paragraph. This is an excellent method. The main drawback is that df/dX may not be easily computed.

2d. Splitting.

In the HIRLAM model we split the physics update from the dynamics update. We also split the various physics sub-processes from each other. There are a number of ways of doing this. In order to discuss them let us abbreviate the equations of motion as $\partial X/\partial t = D_X + P_X$, where D_X and P_X represent the dynamics and physics terms, respectively, and $X = u, v, T, q...$. We integrate the dynamics first :

$$(X^D - X^n)/\Delta t = D_X(X^D, X^n) \tag{2.13}$$

The notation has been chosen to emphasise that some of the terms on the right hand side are integrated implicitly. X^D is the updated value of X after the all of the dynamical computations have been performed. There are now a number of ways of updating the physics.

(a) The physics update proceeds completely independently of the dynamics:

$$(X^P - X^n)/\Delta t = P_X(X^P, X^n) \tag{2.14}$$

$$X^{n+1} = X^n + \Delta t D_X(X^D, X^n) + \Delta t P_X(X^P, X^n). \tag{2.15}$$

X^P is the value of X after updating exclusively by the 'physics'. This is called 'process splitting' by Beljaars (1991).

(b) The physics update 'knows' that the dynamics has changed the model atmosphere. Beljaars (1991) uses the term 'fractional stepping' to describe this procedure. There are a number of options. I will call the procedure described by Eq. (2.16) 'partial fractional stepping' (notice that X^n is used on the right hand side):

$$(X^{n+1} - X^D)/\Delta t = P_X(X^{n+1}, X^n), \tag{2.16}$$

and that described by Eq. (2.17) 'full fractional stepping' (only X^{n+1} and X^D are used):

$$(X^{n+1} - X^D)/\Delta t = P_X(X^{n+1}, X^D). \tag{2.17}$$

Above, we have only discussed the various ways of splitting the dynamics from the physics. However, the same principles apply when we integrate each sub-process within the physics. Should we use total process splitting? Alternatatively, should we include tendencies from the dynamics, and the other, already updated physical processes, when updating a particular process? If the latter, in what order should we perform the physics updates?

Let us examine the implications of the various forms of splitting for the following simple equation

$$dX(t)/dt = (K_1 - K_2)X(t) \tag{2.18}$$

where K_1 and K_2 are constant. The solution is well known: $X(t) = X_0 exp[(K_1 - K_2)t]$. If we integrate this with an Euler explicit scheme .

$$(X^{n+1} - X^n)/\Delta t = (K_1 - K_2)X^n \tag{2.19}$$

then it is stable if $\Delta t(K_1 - K_2) \leq 2$. If this condition holds, but individually $\Delta t(K_1) > 2$ and $\Delta t(K_2) > 2$, then if we split the integration and continue to integrate with the Euler explicit scheme then we would have two unstable components to an overall stable integration. If we now apply process splitting we see that it gives exactly the same result as the unsplit scheme, as does partial fractional stepping. Full fractional stepping, however, is disastrous because the result is dominated by the $O(\Delta t^2)$ terms if $\Delta t(K_1)$ and $\Delta t(K_2)$ are larger than 1:

$$X^{n+1} = X^n[1 + (K_1 - K_2)\Delta t - K_1 K_2 \Delta t^2] \tag{2.20}$$

This simple test indicates that process splitting and partial fractional stepping may be more stable than full fractional stepping if large time steps are being used.

3. OSCILLATION SEEN IN A SEMI-LAGRANGIAN INTEGRATION.

In this section some of the ideas discussed in section 2 are applied to the HIRLAM 2.4 vertical diffusion scheme. The forecast starting from the analysis of 0000 UTC 10 January, 1993 produces oscillations in the lower atmosphere near the centre of the 'world record' area of low pressure if we use the semi-Lagrangian scheme with a time step of 20 minutes and sixth order implicit horizontal diffusion with a coefficient of $K = 1 \times 10^{23}$. The grid spacing of $0.5° \times 0.5°$, and 16 hybrid levels were used in the vertical. The evolution of wind speed at grid point (80,50,14) for the first 24hr of the forecast is shown in fig. 4 by the line joining the 'x's. As was shown in McDonald (1995) this oscillation can be controlled with increased horizontal diffusion. However, here we wish to examine the effect of using different strategies for updating the vertical diffusion scheme on this oscillation.

1. Run the vertical diffusion scheme with a time step of 10mins. while using 20mins. for all the other processes. Interestingly, this did have a positive impact, but did not eliminate the oscillation. See the line joining the boxes in fig. 4. Also shown in this figure is the forecast using a time step of 10mins. for all processes; see the line joining the dots.

2. In the HIRLAM model the time stepping in the vertical diffusion scheme is performed as in the following equation

$$\frac{(X^{n+1} - X^n)}{\Delta t} = \frac{\partial}{\partial z}\left[\gamma \kappa^n \frac{\partial X^{n+1}}{\partial z} + (1 - \gamma)\kappa^n \frac{\partial X^n}{\partial z}\right]. \tag{2.21}$$

with $\gamma = 1.5$. We showed in section 2c that increasing γ should help bring this oscillation under control. Using $\gamma = 4.0$ does have a strong positive impact, without eliminating the oscillation completely. See the triangles in fig. 4.

3. Beljaars (1991) makes strong arguments for using 'fractional stepping'. His general idea is that 'with longer time steps it becomes more and more relevant to keep an accurate balance between processes within a single time step'. Therefore I tested the vertical diffusion scheme in 'full fractional step' mode. (The default in the HIRLAM reference system has been to use 'process splitting'). This change had a remarkable positive impact on the oscillation. See the continuous line without a symbol in fig. 4. Compare it with the highly oscillatory forecast marked by the 'x's. The only difference between these two forecasts is that the former uses 'fractional stepping' when updating the vertical diffusion scheme whereas the latter uses 'process splitting'. (Only the dynamics precedes

the vertical diffusion in the HIRLAM model). I have also integrated the vertical diffusion scheme using 'partial fractional stepping'. The forecast is almost indentical to that shown in by the continuous line without a symbol in fig. 4. So this does not support the argument to the contrary in section 2d.

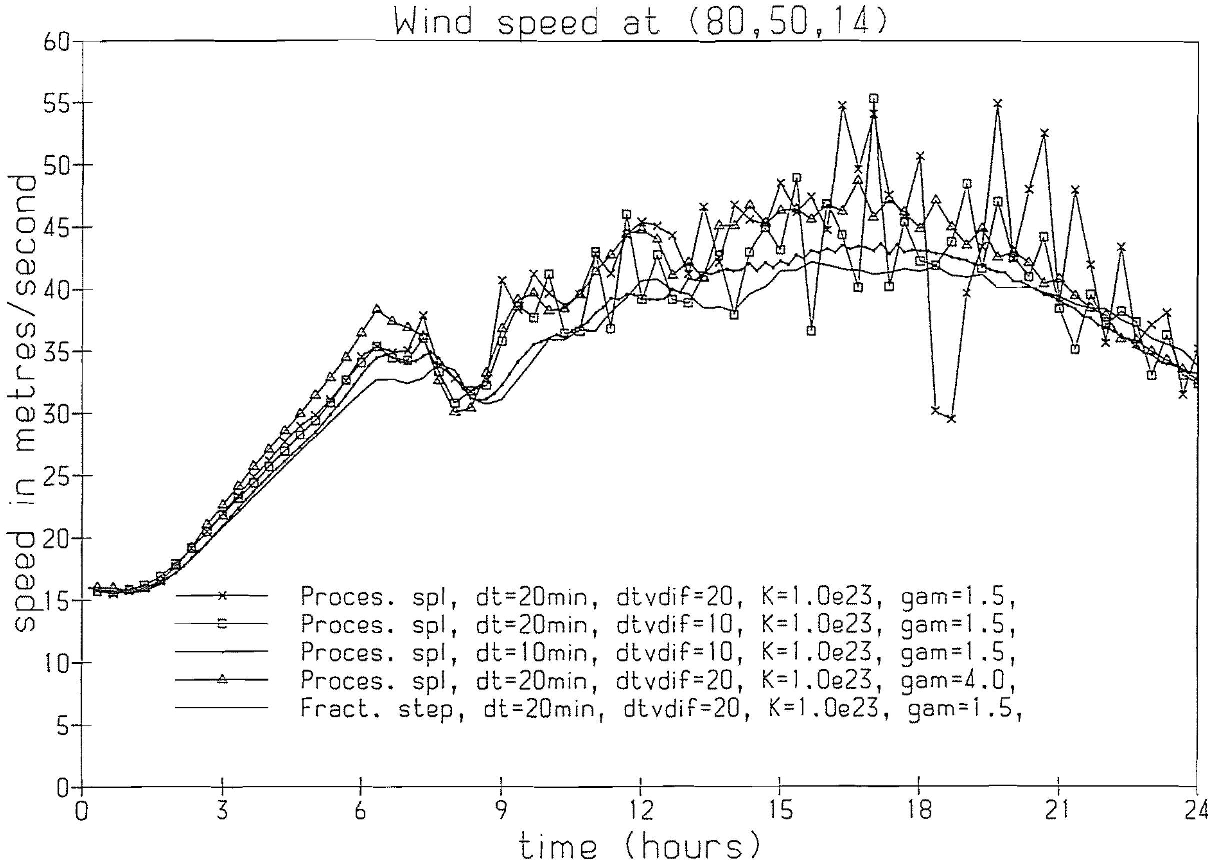

FIGURE 4. Evolution of the model wind speed at location (58N,12W) and vertical level 14.

4. DISCUSSION.

Some simple examples of the kind of problems which may arise when integrating the 'physics' with large time steps were examined.

For stiff systems explicit schemes lead to instability and naive application of implicit schemes can lead to oscillating solutions. The same lesson applies when the system is such that the boundary values differ markedly from the interior values. For both these problematic systems the Euler implicit scheme gave stable and oscillation-free integrations.

The trapezoidal scheme is an $O(\Delta t^2)$ accurate representative of a class of (A stable) schemes which are generally speaking *inappropriate* for solving these kinds of problems. On the other hand, the Euler implicit scheme is the $O(\Delta t)$ accurate representative of a class of (L stable) schemes which *are appropriate* for solving these kinds of problems. The higher order members of the class all seem to be computationally expensive. For a precise definition of A and L stability see, for example, section 6.3 of Lambert (1991).

When non-linear terms appear in the equations then stable schemes which are computationally inexpensive are more difficult to invent. The 'over-implicit' scheme (large γ in Eq. (2.21) is effective. If that fails the method of Eq. (2.12) seems to be the most reliable. Again, both of these schemes are $O(\Delta t)$ accurate, and higher order acccurate versions of them will almost certainly be computationally expensive.

It may be necessary to re-examine the time stepping schemes used in 'physics' with the following ideas in mind. (a) Are we using explicit schemes where implicit ones would be more appropriate? (b) Is the implicit scheme likely to be a source of oscillatory, or even incorrect solutions? (c) In the case of stiff systems, is there a less stiff parameterization?

Splitting is widely used in the HIRLAM model. First the dynamics is split from the physics. Second, all physical parameterization schemes are split from each other. Process splitting is used for most of the individual physical parameterization schemes. In this approach all sub-processes are updated independently of each other. In section 3 we have seen evidence that this may not be the best way to integrate. Beljaars (1991) argued that where certain processes balance each other in the atmosphere then the same balance should be maintained in the numerical model. Janssen et al. (1992) give an example of the importance of including the Coriolis terms in the vertical diffusion update. Closer balance can be most easily attained by replacing process splitting by fractional stepping. It would be interesting to re-examine the HIRLAM physics in light of these arguments.

One way of making the equations more *balanced* would be to integrate both the physics and the dynamics together in an *unsplit* semi-Lagrangian semi-implicit fashion. Two obvious candidates for starting this program are the equations of motion for specific humidity and liquid water. Grabowski and Smolarkiewicz (1996) have shown how to take the first step.

Acknowledgement. Thanks to Erland Källén and Hannu Savijärvi whose constructive comments strengthened the original manuscript.

5. REFERENCES.

Beljaars, A.C.M., 1991: Numerical schemes for parameterizations. *Seminar Proceedings on Numerical Methods in Atmospheric Models. Volume II*, pp 1-42. Available from the European Centre for Medium Range Forecasting, Shinfield Park, Reading, England.

Churchill, R.V., 1963: Fourier series and boundary value problems. *McGraw-Hill* pp 248.

Dalquist, G., 1963: A special stability problem for linear multistep methods *BIT*, **3**, 27-43.

Grabowski, W.W., and P.K. Smolarkiewicz, 1996: Two time level semi-Lagrangian modelling of precipitating clouds. *Mon. Wea. Rev.*, **124**, 487-497.

Janssen, P.A.E.M., A.C.M. Beljaars, A. Simmons, and P Viterbo, 1992: Suitable schemes for non linear vertical diffusion in atmospheric circulation models. *Mon. Wea. Rev.,* **120** , 2977-2985.

Kalnay, E., and Masao Kanamitsu, 1988: Time schemes for strongly nonlinear damping equations. *Mon. Wea. Rev.,* **116** , 1945-1958.

Lambert, J.D., 1991: Numerical methods for ordinary differential systems. *Wiley.* pp 293.

McDonald, A., and J.E. Haugen, 1993: A two time level, three dimensional semi-Lagrangian and semi-implicit grid point model II. Extension to hybrid coordinates. *Mon. Wea. Rev.,* **121**, 2077-2087.

McDonald, A., 1995: The HIRLAM two time level, three dimensional semi-Lagrangian, semi implicit, limited area, grid point model of the primitive equations. *HIRLAM Technical Report no.* **17**.

List of HIRLAM Technical Reports

No.

1. Gustafsson, N., Järvenoja, Kållberg, P. and Nielsen, N.W. (1986). Baseline experiments with a high resolution limited area model. Copenhagen, November 1986.

2. Nordeng, T. E. and Foss, A. (1987). Simulations of storms within the HIRLAM baseline experiment with the Norwegian mesoscale limited area model system. Oslo, March 1987.

3. Gustafsson, N. and Svensson, J. (1988). A data assimilation experiment with high resolution TOVS data. Norrköping, January 1988.

4. Myrberg, K., Koistinen, J. and Järvenoja, S. (1988). A case study of non-forecasted cyclogenesis in polar air mass over the Baltic sea. Helsinki, November 1988.

5. Machenhauer, B. (1988). HIRLAM Final Report. Copenhagen, December 1988.

6. Lynch, P. and McGrath, R. (1990). Spectral Synthesis on Rotated and Regular Grids. December 1990.

7. Kristjánsson, J. E. and Huang, X.-Y. (1990). Implementation of a consistent scheme for condensation and clouds in HIRLAM. December 1990.

8. HIRLAM Workshop on Mesocale Modelling - Copenhagen, Denmark, 3-5 September 1990. December 1990.

9. Gustafsson, N. (1993). HIRLAM 2 Final Report. Norrköping, March 1993.

10. Lynch, P. (1993). Digital Filters for Numerical Weather Prediction. January 1993.

11. Huang, X.-Y., Cederskov, A. and Källén, E. (1993). A Comparison between Digital Filtering Initialization and Nonlinear Normal Mode Initialization in a Data Assimilation System. June 1993.

12. Lynch, P. and Huang, X.-Y. (1993). Initialization Schemes for HIRLAM based on Recursive Digital Filters. October 1993.

13. Eerola, K. (1993). Experimentation with Second and Fourth Order Horizontal Diffusion Schemes. October 1993.

14. Kristjánsson, J. E. and Thorsteinsson, S. (1994). Simulations of intense cyclones near Iceland. Norrköping, March 1994.

15. Sass, B. H. and Christensen, J. H. (1994). A Simple Framework for Testing the Quality of Atmospheric Limited Area Models. Norrköping, August 1994.

16. HIRLAM-2 Radiation Scheme: Documentation and Tests. Norrköping, November 1994.

17. McDonald, A. (1994). The HIRLAM two time level, three dimensional semi-Lagranian, semi-implicit, limited area, grid point model of the primitive equations. Norrköping, March 1995.

18. Gollvik, S., Bringfelt, B., Perov, V., Holtslag, A. A. M. (1995). Experiments with nonlocal vertical diffusion in HIRLAM. Norrköping, March 1995.

19. Bringfelt, B., Gustafsson, N., Vilmusenaho, P. and Järvenoja, S. (1995). Updating of the HIRLAM physiography and climate data base. Norrköping, June 1995.

20. Bazile, E. (1995). Study of a prognostic cloud scheme in Arpege. Norrköping, August 1995.

21. Huang, X.-Y. (1995). Initialization of cloud water content in the HIRLAM data assimilation system. Norrköping, August 1995.

22. Lönnberg, P. Observing system experiments on North Atlantic radiosondes. Norrköping, February 1996

23. Bringfelt, B. Tests of a new land surface treatment in HIRLAM. Norrköping February 1996

24. Lynch, P. A Simple Filter for Initialization, Norrköping, March 1996

25. Perov, V and Gollvik, S. A 1-D Test of a Nonlocal, E-ε Boundary Layer Scheme for a NWP Model Resolution. Norrköping, April 1996

26. Huang, X-Y and Yang, X. Variational Data Assimilation with the Lorenz Model. Norrköping , April 1996

27. McDonald, A. Sources of noise in the "physics"; a preliminary study. Norrköping, November 1996